Complements of Analysis

Teaching language:

Italian

Contents:

Surfaces and integrals on surfaces.

Complements to the theory of ordinary differential equations

Trigonometric series and Fourier series

References:

C.D. Pagani, S. Salsa, Analisi Matematica Vol. 2, Zanichelli
N. Fusco, P. Marcellini, C. Sbordone, *Elementi di Analisi Matematica II*, Ed. Liguori
M. Picone, G. Fichera, *Corso di Analisi Matematica*, Vol. I & II, Ed. Veschi.
M. Tenenbaum, H. Pollard, *Ordinary Differential Equations*, Dover Publications. (In particolare: Ch. 7, p.393-417, 421-423.)
W. Walter, *Ordinary Differential Equations*, Graduate Texts in Mathematics, Springer. (In particolare: Ch. 1 (tranne Sec. XIV, p.24-27, Supplement p.33-35; Sec. VI, p.41-45); Ch. 4, Sec. 17, p.175-189).

General educational goals:

The course intends to provide the basic notions on surfaces, calculus of integrals on surfaces, ODE and related Cauchy problems systems of ODE, trigonometric series and Fourier series. The student shall have to gain steady theoretical expertise and to know how using methods and concepts in order to solve exercises and problems.

Prerequisite:

Basic knowledge of analysis, with particular reference to the contents of standard courses of Analysis I and Analysis II

Educational approach:

Traditional lesson on the blackboard

Exam:

Written exercises and oral examination

Contents in details

Regular simple surfaces. Tangent plane and normal line at a point of a regular simple surface. Implicit form. Cartesian surfaces. Orientation. Schwarz phenomenon. Minkowski's definition of area. Rotation surface, Guldino's theorem. Integrals on a surface. Differential forms of degree two and related integral. Stokes' Theorem. Orientable surfaces: Moebius strip. Gauss-Green formulas. Divergence theorem. The equation rot u = v.

Metric spaces and their completeness. Banach-Caccioppoli theorem. Completeness of $C(K)$. Local and global existence and uniqueness theorem for systems of ODE in normal form. General integral, particular integral, singular integral. Global existence of the general integral theorem. Gronwall's lemma and the continuous dependence from the initial data in the Cauchy problem.
Complements on linear equations: variation of the constants method (Lagrange). Study of particular types of equations: separable variables, of the type $y'=f(ax+by+c)$, $y'=f(y/x)$, $y'=f\left(\frac{ax+by+c}{a'x+b'y+c'}\right)$, $y'+g(x)y+h(x)y^{\alpha}=0$, $\alpha\neq1$(Bernoulli), $y'+g(x)y+h(x)y^{2}=k(x)$ (Riccati), $x=g(y')$, $y=g(y')$, $y=xy'+g(y')$ (Clairaut), $y=xf(y')+g(y')$ (d'Alembert), $f(x,y',y'')=0$
Cauchy problem related with ODE of order n in normal form, Local existence and uniqueness theorem. ODE of second order: of the type $f(y,y',y'')=0$, $f(x,y,y',y'')=0$ with f homogeneous in (y,y',y''), Eulero equation. The equation $X(x,y)dx+Y(x,y)dy=0$: integrating factor method.
Systems of ODE. Nondegenerate linear systems . Theorem on the number of arbitrary constants. Degenerate linear systems.

Trigonometric series. Fourier coefficients of a generally continuous summable functions. Fourier series. An approximation lemma. Riemann-Lebesgue lemma. Riemann localization principle. Dirichlet kernel. Dini's convergence Theorem.
Cesàro's theorem on the sequences. To sum according to Cesàro. A sufficient condition for the ordinary convergence of a series converging according to Cesàro. Hardy's theorem (without proof). Fejér kernel. Fejér's convergence theorem. A sufficient condition for the uniform convergence of Fourier series. Weierstrass' approximation theorem.

Final remark:

The course aims to develop basic knowledge in analysis and to promote the comprehension of the logical precision in the proof and the capacity to elaborate autonomously a proof.