Course: Mathematical Analysis I

Teacher: Prof. Paolo Vitolo
(see Curriculum Vitae of teacher)

Teaching Language: Italian

Contents:
Real numbers. Functions and graphs.
Continuity and uniform continuity. Limits.
Induction principle. Sequences. Cauchy criterion.
Subsequences; theorem of Bolzano-Weierstrass.
Heine-Cantor theorem. Weierstrass Theorem.
Derivative. Derivation rules, derivatives of elementary functions.
Derivative of a monotone function. Fermat's theorem, Rolle's theorem.
Theorems of Lagrange and Cauchy. Theorem of De L'Hôpital.
Study of functions.
Riemann integration. Improper integrals.

Textbooks:
E. Giusti: Analisi Matematica I, Bollati Boringhieri.
E. Acerbi, G. Buttazzo: Primo corso di Analisi Matematica, Pitagora Editrice.
E. Giusti: Esercizi e Complementi di Analisi Matematica, vol. 1, Bollati Boringhieri.
G. Gilardi: Analisi Uno, McGraw-Hill.

Educational Goals:
Understand the basic concepts of Mathematical Analysis.
Learn the tools of differential and integral calculus for functions of one real variable.
Know how to solve some basic types of inequalities.
Calculate limits of functions or sequences.
Draw a qualitative and quantitative representation of the graph of a function.

Prerequisites:
Some basic knowledge of plane geometry. Equations and inequalities of first and second degree.

Teaching methods:
The learning objectives will be achieved both through lectures and through exercises.

Assessment methods:
A written test and an oral test.

Extended program:
Axioms of the real numbers. Upper and lower bounds, maximum and minimum; supremum and infimum, the upper bound principle. The extended real and intervals.
Nth root of a real number; power with real exponent; logarithms.
Well-ordering of the natural numbers; induction principle. Definitions by recurrence.
Property of Archimedes. Integer part of a real number. Trigonometric functions: sine, cosine and tangent; trigonometric formulas.
Absolute value; triangle inequality.
Functions and graphs. Increasing functions and decreasing functions. Operations with functions. Inverse of a function; invertibility of the functions that are strictly monotone. Inverse trigonometric functions.
Density of the rationals and of the irrationals.
Continuity and uniform continuity. Operations with continuous functions.

Bernoulli's inequality. Trigonometric inequalities. Continuity of elementary functions. Examples of continuous functions which are not uniformly continuous.
Neighbourhoods. Theorem of the permanence of the sign. Accumulation points and isolated points. Theorem of the zeros; intermediate value theorem.
Limit of a function at a point; uniqueness of the limit. Operations with limits. Limit comparison theorem. Limits of restrictions; limits from the right and from the left. Limits of monotone functions. Infinite limit; indeterminate forms. Limit of a function at inifinity.
Limit of the composition of two functions. Connection between continuity and limits of functions. Fundamental limits of elementary functions. Classification of points of discontinuity; discontinuity of monotone functions. Criterion of continuity for monotone functions.
Sequences. Meaning of "eventually" and "frequently." Theorem of the permanence of the sign and limit comparison theorem for sequences. Characterization of accumulation points by means of sequences. Connection between limits of functions and limits of sequences.
Characterization of monotone sequences. Construction of the number e. Basic limits. Subsequences; theorem of Bolzano-Weierstrass. Cauchy convergence criterion. Limsup and liminf of a function. Characterization of the limit by means of limsup and liminf. Limsup and liminf of a sequence.
Asymptotic comparison of functions. Notations "o" and "O". Infinitesimals and infinities.
Open and closed sets. Weierstrass Theorem. Heine-Cantor theorem.
Points of local maximum and minimum. Characterization of invertible continuous functions. Continuity of the inverse of a continuous function.
Definition and geometrical meaning of the derivative; higher order derivatives.
Differentiable functions. Continuity of differentiable functions. Equivalence between differentiability and differentiability. Derivation rules, derivatives of elementary functions. Examples of functions that are not differentiable.
Fermat's Theorem. Rolle's theorem. Cauchy's theorem. Lagrange's theorem.
Increasing or decreasing character of a function. Convex functions and their properties; inflection points. Asymptotes. Study of functions.
De L'Hôpital theorem and its applications.
Riemann integrable functions; integral of a function on an interval. Example of a non integrable function.
Linearity of the integral. Integral of a nonnegative function. Monotony of the integral. Integrability criterion.
Integrability of monotone functions. Integrability of composite functions. Integrability of continuous functions. Integrability of the product of two integrable functions. Restrictions and extensions of integrable functions. Segmental property of the integral. The mean value theorem; the generalized mean value theorem.
Definite integral. Integral function and its properties. The fundamental theorem of calculus.
Primitive of a function, the indefinite integral. Calculation of the definite integral; examples. Immediate integrals. Methods of indefinite integration. Integration of rational functions; Hermite formula. Integration by rationalization.
Improper integral: definition and examples. Integrable functions and summable functions. The comparison criterion for summability. Summability criteria. Example of an integrable function which is not summable.
Taylor expansion. Remainder of the Taylor expansion: the form of Peano and the form of Lagrange. Integral form of the remainder of the Taylor expansion.

Notes:
This course contributes primarily to provide students with the basic knowledge of Mathematical Analysis, improve the ability to do rigorous proofs, and understand the basic applications of mathematics to other sciences.