**Numerical Treatment of Functional Equations – Module B**

**Language**
Italian

**Contents**
- Approximation of non-periodic functions by means of algebraic polynomials on bounded intervals of the real line
- Numerical methods for the global approximation of the solution of Fredholm integral equations of the second kind on bounded intervals having non-periodic given functions

**Books**
- Lecture notes in pdf form (Italian)

**Course goals or objectives**
The aim of the course is to know and to apply stable and convergent numerical methods for the global approximation of the solution of some functional equations (in particular, Fredholm integral equations of the second kind).
Moreover, the course will give the Approximation Theory tools for the construction of the numerical methods. In particular, in Module B, the approximation of non-periodic functions by means of algebraic polynomials will be considered.
By the end of Module B students should be able to:
- understand the basic ideas of the theory of weighted polynomial approximation;
- connect structural properties of a non-periodic function with the order of convergence of its best weighted polynomial approximation;
- know the behavior of polynomial approximation processes, such as Fourier sums and Lagrange interpolation, in different function spaces with weighted metric;
- know and apply numerical methods for the global approximation of the solution of functional equations having non-periodic given functions;
- discuss the stability and the convergence of these numerical methods;
- implement algorithms related to these numerical methods and comment the numerical results, comparing them with the theoretical estimates.

**Prerequisites**
- the topics of Module A of this course
- mathematical analysis (differential and integral calculus for functions of one or several variables, sequences and series, normed spaces, linear operators, elements of complex analysis)
- linear algebra and geometry (linear spaces, linear systems, linear maps, eigenvalues, orthonormal bases)
- numerical analysis (errors, quadrature rules, linear systems)
- elements of computer programming (MatLab)
Teaching methods

- Lectures

The active participation of the students will be encouraged during the lectures. Then, the students will have the possibility to take part to mid-course oral and/or practical tests to check their learning level.

Means of evaluation

Oral exam at the end of the course

Detailed contents

- **Function spaces with weighted metric.** Weight functions and weighted polynomial approximation. Spaces of continuous functions with weighted $L^p$–norm. $q$–Moduli of smoothness, $K$–functionals and their properties. Sobolev-type and Zygmund spaces.

- **Algebraic polynomials.** Some properties of algebraic polynomials. Orthogonal polynomials with respect to a weight function. Weighted polynomial inequalities: Bernstein, Markov and Nikolskii inequalities.


- **Quadrature rules.** Error estimates of the gaussian quadrature rule in Sobolev-type spaces.