COMPARATIVE STUDY OF STAGNATION POINT ANOMALIES BY MEANS OF SHOCK CAPTURING AND SHOCK FITTING UNSTRUCTURED CODES

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ABSTRACT

The two-dimensional, hypersonic ($M_{\infty} = 17.605$), laminar flow ($Re_{\infty} = 376930$) past the forebody of a circular cylinder has been simulated by means of a vertex-centred CFD code using linear triangular elements. Two different approaches have been used to simulate the strong detached bow shock: shock-capturing on anisotropically refined meshes and shock-fitting. Concerning the boundary layer mesh, the distribution of gridpoints has been kept constant, while three different connectivity patterns have been examined. When looking at wall quantities such as pressure, skin friction and heat transfer these appear to be more heavily affected by the boundary layer mesh than by the numerical model used to simulate the detached shock wave.

Key words: shock-capturing, shock-fitting, unstructured triangular grids.

1. INTRODUCTION

Unstructured-grid CFD codes are nowadays being considered as the future computational tool for the aerothermodynamic simulation of hypersonic flows. It is indeed widely acknowledged that unstructured grids provide “the greatest flexibility to adapt to evolving flow structures .. and deforming bodies ... without a requirement for significant user intervention” [1]. To date, however, the accuracy delivered by unstructured grid codes in hypersonic applications is markedly inferior to that exhibited by their structured-grid counterpart. As documented in a recent AIAA Meeting [2, 3, 4, 5, 1], this appears to be particularly true when triangular/tetrahedral elements are used throughout the computational domain. These deficiencies manifest themselves (among others) as over-predicted heat rates and anomalous recirculation bubbles in the surrounding of the stagnation point. Although the use of mixed type elements might alleviate the problem, quad/hex cells being used within the boundary layer and to capture the bow shock, it is also true that simplicial elements (triangles and tetrahedra) greatly simplify automatic grid generation and feature-based adaptation.

In this paper we try to establish to which extent shock and boundary-layer resolution contribute to the observed stagnation point anomalies. It is indeed a common belief that triangular/tetrahedral cells perform badly both in capturing shock waves and within boundary layers, because of the presence of an edge which is necessarily unaligned with the dominant flow direction. This is schematically shown in Fig. 1.

In order to discriminate between these two possible causes, we have conducted a number of numerical experiments keeping the distribution of gridpoints within the boundary layer grid fixed while changing the mesh connectivity and using two very different approaches to model the detached shock wave: shock capturing (SC) on anisotropically refined grids and shock fitting (SF).

The chosen testcase had originally been proposed by the NASA Langley aerothermodynamic team in the framework of the High Energy Flow Solver Synthesis (HEFSS) [6]. It consists in the two-dimensional, hypersonic, laminar flow of air past the forebody of a circular cylinder. Beside the reference results obtained using the NASA codes LAURA (structured-grid) and FUN2D (unstructured-grid), the only other references known to the authors which deal with the present testcase are those of Wood [7], who uses both a Finite Volume and a Fluxion Splitting code and Quattrochi [8] who uses a Discontinuous Galerkin code.

Figure 1. Schematic of the mis-alignment problem encountered with triangular elements when simulating a shock wave or a boundary layer flow.
2. COMPUTATIONAL TOOLS

In the present study, shock modeling has been accomplished by means of two different approaches: shock-capturing on anisotropically refined meshes and shock fitting. In both cases the gasdynamic solver is EulFS [9], a 2D/3D unstructured, vertex-centred code using triangular/tetrahedral elements with continuous, piecewise linear data representation. EulFS relies upon fluctuation splitting schemes for the spatial discretization and implicit pseudo time-stepping to accelerate convergence towards steady state. When the bow shock is captured, a finite number of grid levels are generated by repeated application of the anisotropic mesh refinement (AMR) algorithm described in [10] and implemented in the freely available software ANGENER [11]. When the bow shock is fitted, a time-evolving grid is used in which the shock is discretized by means of a polygonal line and treated as a boundary of zero thickness interior to the computational domain. The motion of the shock front and the state values within its upstream and downstream nodes are computed according to the Rankine-Hugoniot equations. Due to the displacement of the shock wave, the mesh in the neighbourhood of the shock front needs to be regenerated at each time-step to guarantee that the shock points and edges are part of a constrained Delaunay triangulation covering the entire computational domain. When applied to steady problems, the shock front will eventually reach its final (steady) position, so that the gridpoint location and cell connectivity no longer change and the shock-fitting grid reverts to a fixed grid. In the present work, the shock-fitting grids have been built upon the un-adapted (level 0) grids used in shock-capturing mode and therefore differ from these only in the neighbourhood of the shock front. Figure 2 should illustrate well the differences between the anisotropically adapted mesh and the shock-fitting one: the fitted shock is shown by means a solid black line and overlayed upon the anisotropically adapted mesh. Further details concerning the shock-fitting algorithms can be found in [12].

3. PROBLEM STATEMENT AND SIMULATION PARAMETERS

The chosen testcase had been proposed in the past by the aerothermodynamic team working at NASA Langley. It consists in the two-dimensional, hypersonic, laminar flow past the forebody of circular cylinder. Since the main focus of this study is upon CFD modeling issues, air is considered as a calorically and thermally perfect gas, with constant specific heats ratio, $\gamma = 1.4$. The freestream conditions are summarized in Table 1. All grids used in the present study have been derived from a structured quadrilateral grid consisting in 64 CVs in the wall normal direction and 60 CVs along the semi-circular forebody surface. This structured quadrilateral mesh was made available along with the benchmark results and had been optimized using some of the features of the structured-grid simulation code LAURA. These include the capability to align the outer (inflow) boundary with the captured bow shock and also to optimize the distribution of grid points in the boundary layer region. Starting from the LAURA mesh, three sequences of grids have been obtained by cutting each quadrilateral cell into two triangles using three different triangulation patterns which we shall refer to in the following as triangulation patterns of type 1, 3 and 6. A detail of the three types of un-adapted (level 0) grids is displayed in Fig. 3, in which the freestream flow is from left to right. In the triangular grid provided by the Langley benchmark, also shown in Fig. 3, all quadrilateral cells had been cut with uniform bias. Since the mesh covers the whole forebody of the cylinder, the Langley triangular grid, which we have labeled type 1, is not symmetric with respect to the direction of the freestream flow, see Fig. 3(a). This was deliberately done so as to emphasize grid induced anisotropies in the computed solution. Beside the Langley’s triangular grid (type 1), we have also considered symmetric triangulation patterns which we have labeled type 3 and 6, see Fig. 3(b) and 3(c).

For each of these three types of triangulation patterns, the subsequent grid levels share with the level 0 grids the same meshpoint location and cell connectivity of the 40 layers of cells closest to the walls. This was done following the choice made in [8]. Outside of this near-wall layer of cells, which encloses the boundary layer, the computational domain has been re-meshed using the anisotropic mesh refinement algorithm (AMR) described in [10]. These two sub-meshes are then glued together. This ensures that, for a given triangulation pattern, the

<table>
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<th>Value</th>
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<td>$V_\infty$</td>
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<tr>
<td>$\rho_\infty$</td>
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<td>$T_w$</td>
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<td>$Re_\infty$</td>
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Table 1. Freestream conditions for the hypersonic 2D cylinder flow.
“boundary layer” meshes are identical for all grid levels. This should be well clarified by Fig. 2. Observe that on grid level 0, the location of the gridpoints (as well as the connectivity for grid types 3 and 6) is symmetric w.r.t. to the stagnation streamline, i.e. the \( y = 0 \) line in Fig. 3. Concerning the subsequent grid levels, symmetry is retained only within the boundary layer mesh, since the adaptation procedure is applied to the whole domain, regardless of the presence of a symmetry axis.

Concerning mesh generation in the SF calculations, the original triangulation pattern within the boundary layer region is retained up to a distance from the wall equal to \( \delta / R = 210^{-3} \). Outside this inner layer of cells, a constrained Delaunay triangulation is built using the meshpoints of the level 0 grids and the shock points and edges that make up the fitted shock front. It should be stressed again that, for a given triangulation pattern, the shock-fitting grid and all levels of the AMR grids are identical within the boundary layer region.

Table 2 summarizes and compares, for each of the three types of boundary layer triangulation patterns, the characteristics (number of gridpoints and cells) of the various grid levels of the AMR and shock-fitting grids. Ideally, the AMR algorithm should converge towards a “nearly” constant number of gridpoints and cells as the grid level is increased. However, for the purpose of the present analysis, 3 or 4 grid levels are seen to be sufficient to obtain nearly grid-converged solutions. Concerning the SF grids, Tab. 2 shows that these are of comparable size as the un-adapted (level 0) shock-capturing grids. This is explained by the observation that these two grids only differ in the neighbourhood of the fitted shock wave. Concerning the spatial discretization, both types of calculations have been performed using a non-linear blending of two linear schemes: the first order accurate, monotone N scheme and the second order accurate, non-monotone LDA scheme. When operating in SF mode, calculations have been repeated using linear, second-order accurate schemes throughout the whole flow-field which is shock-free since the only shock is fitted. For a given triangulation pattern, the differences observed among the various linear schemes and the non-linear (blended) scheme were seen to be negligible.

4. COMPUTATIONAL RESULTS

In this section we present the computational results obtained using the three different triangulation patterns for the boundary layer meshes. For each of these, a comparative assessment is made between the results obtained by using the shock-capturing and shock-fitting models to simulate the bow shock. The comparison is primarily conducted by looking at wall quantities, in particular: pressure coefficient, skin friction coefficient and heat transfer rate, since these enter directly into the calculation of the aerodynamic forces and thermal load acting on a space vehicle re-entering the atmosphere.

As a preliminary step, we have conducted a qualitative assessment of the relative merits of the capturing and fitting models by analyzing the vorticity field computed downstream of the bow shock, within an essentially inviscid region of the flow-field. This was motivated by a similar study [13] and also by the reading of [14] where it is stated that: “Conditions at the boundary-layer edge in turn, particularly in the stagnation region of hypersonic flows, are dependant on entropy carried along streamlines from the shock. Any irregularities in the captured shock create associated irregularities in entropy that feed the rest of the domain”. Figure 4 shows the normalized vorticity profile extracted along a line perpendicular to the stagnation streamline and located 0.2 radii (at \( x = -0.2 \) in Fig. 2) ahead of the stagnation point, a distance which is approximately equal to half of the shock stand-off distance. The vorticity profile shows two evident jumps due to the intersection of the rake with the bow shock while it varies almost linearly in the post-shock region. The three profiles displayed in Fig. 4 are the one computed by means of shock-fitting and those computed in shock-capturing mode on the un-adapted level 0 grid and the AMR level 4 grid. Observe that: i) strong oscillations oc-
Table 2. Characteristics of the grids used in SC and SF mode.

<table>
<thead>
<tr>
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<tr>
<td>fitting</td>
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Even though this preliminary analysis indicates that a sharper shock resolution, either by SF or AMR, improves the quality of the flow-field just ahead of the stagnation region, these improvements appear to have little effect upon the quantities computed on the wall, as will be detailed in the following sub-sections.


We shall examine first the symmetric triangulation patterns type 3 and 6, respectively shown in Figs. 3(b) and 3(c). The wall distribution of the pressure coefficient is displayed in Figs. 5 and 6 while the skin friction coefficient is displayed in Figs. 7 and 8. All wall quantities are plotted against the azimuthal angle $\Theta$ which takes 0 value at the stagnation point. The various curves shown in these plots include the shock-capturing solutions obtained on grid levels 0, 2, 3 and 4, the shock-fitting solution and the LAURA solution, which is taken as the reference.
reference one. The following observations can be made. For a given triangulation pattern, the solutions obtained in shock-capturing mode appear to be in mutual close agreement starting at grid level 2; moreover, the shock-fitting solution and the shock-capturing solution computed on the last grid level also show small mutual differences. However, when comparing wall data computed using the two different triangulation patterns, far larger differences are observed. This is particularly true for the skin friction coefficient, see Figures 7 and 8. These observation suggests that the wall distributions are more heavily affected by the location of gridpoints and mesh connectivity within the boundary layer region than by the numerical model used to simulate the bow shock. Moreover, in the case of the symmetric triangulation patterns we are examining, a relevant (and detrimental) role appears to be played by the presence of the symmetry axis. Since we are using a vertex-centred code, and the triangular grids originate from a structured one within the boundary-layer, a number of gridpoints are indeed located precisely on the symmetry axis. The presence of a “symmetry-axis problem” can be clearly seen in the pressure coefficient distribution, which shows a clear spike in the neighbourhood of the stagnation point for the triangulation pattern of type 3 and a similarly evident dip for the triangulation pattern of type 6. The amplitude of the center-line dip/spike is reduced by improving the shock resolution (by means of either AMR or SF), but not completely removed. The “symmetry-axis anomaly” can also be clearly seen in Figs. 9 and 10 where the pressure distribution along the stagnation streamline is plotted against the distance from the (inviscid) stagnation point. To emphasize the boundary-layer region, the abscissae have been plotted in logarithmic coordinates. It is clearly seen that the present results overshoot the reference value on the triangulation pattern 3 and undershoot on the triangulation pattern 6. This anomalous behaviour appears to be closely linked to other unexpected flow features. In the case of the type 3 triangulation, reverse flow appears on both sides of the stagnation streamline. This is evident from the skin friction profile of Fig. 7 which changes sign on both sides of the stagnation point. On the un-adapted (level 0) grid this effect is so pronounced that a large region of recirculating flow arises. Improving the resolution of the shock wave, either by AMR or SF, reduces considerably the extent of the recirculation bubble, but fails to remove it completely. This anomalous boundary layer development in the neighbourhood of the stagnation point considerably affects the skin friction profile on the remainder of the body. On the type 6 grids, there is no flow reversal along the body, see Fig. 8, but a small pocket of reverse flow appears within the gridpoints closest to the wall which are located on the stagnation streamline.

4.2. Un-symmetric triangulation pattern type 1.

The triangulation pattern type 1, shown in Fig. 3(a) is the unstructured triangular grid of the Langley benchmark. Uniform subdivision of the quadrilateral cells into triangles leads to a grid which is un-symmetric w.r.t. to the $y$ symmetry axis. Grid-convergence of the SC calculations for this triangulation pattern appears more problematic than in the case of the symmetric triangulations reported in the previous section. This is evident when looking at the wall distribution of pressure coefficient computed on the various grid levels in shock-capturing mode and displayed in Fig. 11. The discrepancies observed among the
solutions computed on the various grid levels are partly attributable to the non-negligible level of un-steadyness which occurs on all grid levels featuring the type 1 triangulation pattern within the boundary layer region. A convergence history representative of this behaviour is displayed in Fig. 13, which refers to grid level 4. On this triangulation pattern, the SF calculation is seen to perform slightly better than the SC calculations. This can be seen, for instance, when looking at the pressure coefficient distribution computed in SF mode which is plotted in Fig. 12 against the LAURA reference. When looking at the skin friction distribution, however, the differences between the SC and SF solutions are not that significant, particularly in the neighborhood of the stagnation point, see Fig. 14. On this type of boundary layer grid the location of the stagnation point displays a considerable offset (approximately equal to 8°) w.r.t. the correct position; also the maxima reached on both sides of the stagnation streamline are considerably different. Figure 14 closely resembles Fig. 7 of [14]; although in the latter case the simulation was for a five species model in chemical nonequilibrium and thermal equilibrium. It should be mentioned that the FUN2D calculation (not shown here) on the type 1 mesh shows a less pronounced asymmetry.

Figure 10. Pressure coefficient distribution along the stagnation streamline on the grids with triangulation pattern of type 6

Figure 11. Pressure coefficient computed in SC mode on the various grid levels with triangulation pattern of type 1

Figure 12. Pressure coefficient computed in SF mode on the grid with triangulation pattern of type 1

Figure 13. Residual convergence history on grid level 4, triangulation pattern of type 1

Figure 14. Skin friction coefficient computed on the grids with triangulation pattern of type 1
than that observed in Fig. 14.

Finally, Fig. 15 shows the most disconcerting result: the normalized surface heat flux computed by means of SF and SC for all types of triangulation patterns. The SC calculation displayed is that of the finest grid level available. One observes that, for a given triangulation pattern, SF and SC give similar trends, but, particularly in the neighborhood of the stagnation point, the predicted heat flux shows significant quantitative differences. Consistently with similar studies [7, 8], the computed heat flux overestimates the value predicted by the structured-grid code LAURA.

At this stage we might be lead to conclude that, regardless of how accurately is the bow shock simulated, the computed wall quantities heavily depend on the boundary layer mesh and, in particular, upon the mesh connectivity. The heavy role played by the triangulation pattern upon the computed solution has also been highlighted by Wood in a very similar context, see [7], pag. 168. Indeed, when we compare the skin friction coefficient computed using the three types of boundary layer triangulation patterns, Figs. 7, 8 and 14 clearly show that there exist huge differences among the solutions computed using the three different boundary layer grids. On the other hand, for a given triangulation pattern, the differences between the SF and AMR solutions are far smaller.

No such clear-cut conclusion can be draw when looking at the heat flux, Fig. 15, since in this case the quantitative differences among the various grids and modeling practices (SF vs. SC) are of comparable magnitude.

The strong grid-dependence exhibited by triangular (and tetrahedral) elements seems to suggest that these type of element shapes are particularly unsuited for boundary layer flows or that the currently available discretization schemes perform very poorly when applied on triangles and tetrahedra. This is a quite common belief, shared by many authors, see e.g. [5].

We think, however, that this does not tell the whole story. We have next considered the stagnation point flow that arises when an incompressible stream impinges perpen-

dicularly on a flat surface. This is a flow configuration for which an exact solution of the incompressible Navier-Stokes equations, due to Hiemenz, is known. The simulation has been conducted at the same Reynolds number as the hypersonic cylinder flow and using the same un-adapted (level 0) grid, flattened against the plate. The same FS code and discretization schemes have been used, although the numerical model in this case is represented by the pseudo-compressibility formulation of the isothermal, incompressible Navier-Stokes equations. Figure 16 shows the skin friction coefficient computed on both sides of the stagnation point (located at $x = 0$ in Figure 16) using all three types of triangulation patterns and compared against the analytical Hiemenz solution. It can be observed that the numerical results computed on the three grids do indeed show a certain amount of grid dependence, but none of the bizarre features displayed in the hypersonic case. The analytical curve falls between the two curves computed using the symmetric triangulation patterns 3 and 6 while the result computed on the asymmetric grid switches between the former two across the stagnation line, as one would expect. It is true, of course, that the incompressible Hiemenz flow is an oversimplified model of the stagnation region in the hypersonic cylinder case. In particular, the thin temperature boundary layer that brings the post-shock temperature down to the cold wall value is missing completely.

Wall curvature is another ingredient which is missing in Hiemenz flow. The importance of a correct numerical description of wall curvature is advocated in [8] where the same hypersonic test case is examined by means of a Discontinuous Galerkin Finite Element code. We have however examined the hypersonic flow against a flat nosed body at the same freestream conditions of Table 1 and noticed the same type of anomalies observed for the cylindrical geometry.
5. CONCLUSIONS

In this paper we believe to have shown that many, if not most, of the devils that show up in the simulation of hypersonic viscous flows on unstructured grids are hidden within the boundary layer, rather than in the shock wave. Before dismissing triangular/tetrahedral elements as unsuited for boundary layer simulations, a more thorough understanding of the performances of this type of elements under hypersonic flow conditions is deemed necessary. To this end, flow configurations for which analytical results are available should be carefully examined for a wide range of freestream parameters and boundary conditions. Flat plate boundary layer flows are the obvious candidates for this purpose, but not the only ones available. The compressible version of Heimenz flow could prove useful towards this end. Boundary conditions is another area in which due care should be exerted, especially under extreme flow conditions.

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