A new approach to the confinement of R/C columns

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ABSTRACT: A new constitutive law is presented, valid for confined concrete. The transversal stresses induced by a hoop, either of square or of circular shape, in the cross section of r.c. members (columns or beams) axially loaded, are evaluated through Airy’s functions relevant to plain strain states. The results, valid for square or circular hoops, are then extended to hoops of polygonal shape, with or without bindings, and to a combination of hoops of different shapes. The formulation, valid for the cross section containing the hoop, is finally extended to the overall volume of the member through the interaction among hoops and longitudinal reinforcements. The results suggested by the proposed model for members of circular and square cross section are compared with experimental data and with the results of other researchers. The proposed model, in comparison with other models, shows a better agreement with experimental results.

1 INTRODUCTION

The classical confinement theory neglects the flexural stiffness of the confining reinforcement and assumes the sides of each hoop behaving like ties between corners of the member cross section. The forces applied at the corners by ties are obtained via equilibrium equations. They are assumed to be distributed by an arching action, from corners within the section, and to be constant at any level of axial strain. An upper bound of the confinement pressures has been so obtained, when the stress in the transversal steel joins yield. These assumptions are not always realistic, when increasing the ratio between diameter of hoops and their flexural length. In reality, to a low axial strain correspond very little transversal strains, in the concrete and negligible stresses, in the confining steel; the concrete is, therefore, unconfined. When the axial stress in the concrete gets closer to the axial compressive strength, the transversal strains increase sensibly; the transversal steel gives a passive confinement, thus. The proposed passive confinement model follows strictly the variations of the transversal strain; it guarantees both equilibrium and displacement compatibility between hoops and concrete core and a distribution of stresses within the cross section depending on the concrete axial strain.

2 PROPOSED MODEL FOR PASSIVE CONFINEMENT

It can be assumed that the increase $\Delta \sigma_z$ of concrete axial stress $\sigma_z$ due to confinement respects plain strain conditions. In fact, given an axial strain, the total axial stress equals the sum of unconfined axial stress $\Delta \sigma_z = \nu (\sigma_x + \sigma_y)$ and of axial stress due to confinement $\Delta \sigma_z$, which appears without any further axial strain ($\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$). In plain strain states (Franciosi 1985, Giangreco 1985), when $\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$, the strain and stress axial components (orthogonal to xy plane) on concrete are:

$$\varepsilon_z = \frac{1}{E} (\Delta \sigma_z - \nu (\sigma_x + \sigma_y)) = 0 \quad \Rightarrow \quad \Delta \sigma_z = \nu (\sigma_x + \sigma_y)$$

(1)

If mass forces are constant or null, given an Airy’s function $f(x, y)$ (introduced by G.B Airy in 1862), the stresses are expressed by (2). The (2) satisfy internal equilibrium, while compatibility is
imposed through (3) and equilibrium at the boundaries of a square shaped cross section trough (4). The direction cosines of the boundary normal are \( \alpha_x, \alpha_y \), the boundary stresses are \( p_x, p_y \).

\[
\sigma_x = \frac{\partial^2 f}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 f}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 f}{\partial x \partial y} \tag{2}
\]

\[
\frac{\partial^4 f}{\partial x^4} + \frac{\partial^4 f}{\partial x^2 \partial y^2} + \frac{\partial^4 f}{\partial y^4} = 0 \tag{3}
\]

\[
\sigma_x \alpha_x + \tau_{xy} \alpha_y = p_x; \quad \tau_{xy} \alpha_x + \sigma_y \alpha_y = p_y \tag{4}
\]

For the \( f(x,y) \) of a square shaped section the present model suggests the two expressions (5) and (6):

\[
f_2(x,y) = \frac{C_{20}}{2} X^2 + \frac{C_{02}}{2} Y^2 \tag{5}
\]

\[
f_4(x,y) = \frac{C_{40}}{4} X^4 + \frac{C_{22}}{2} X^2 Y^2 + \frac{C_{04}}{4} Y^4 \tag{6}
\]

Imposing the boundary conditions (4), the applied boundary stresses \( p_x, p_y \) are obtained, so satisfying equilibrium. Both (5) and (6) agree with (3); trough (2) they give the stress components expressed by (7) and (8), respectively. Due to (3), (6) gives \( C_{22} = -(2C_{22} + C_{04}) \).

\[
\sigma_x = \frac{\partial^2 f}{\partial y^2} = C_{02}, \quad \sigma_y = \frac{\partial^2 f}{\partial x^2} = C_{20}, \quad \tau_{xy} = -\frac{\partial^2 f}{\partial x \partial y} = -2 C_{22} XY \tag{7}
\]

\[
\sigma_x = \frac{\partial^2 f}{\partial y^2} = C_{22} X^2 + C_{04} Y^2; \quad \sigma_y = \frac{\partial^2 f}{\partial x^2} = C_{40} X^2 + C_{22} Y^2; \quad \tau_{xy} = -\frac{\partial^2 f}{\partial x \partial y} = -2 C_{22} XY \tag{8}
\]

Putting \( C_{20} = C_{02} = -B l^2 \) in (5), the stress state (9) is obtained (see fig. 1), while assuming \( C_{22} = A \) and \( C_{40} = C_{04} = -A \) in (6), the stress state (10) is obtained (see fig. 2).

\[
\sigma_x = \frac{\partial^2 f}{\partial y^2} = -B l^2, \quad \sigma_y = \frac{\partial^2 f}{\partial x^2} = -B l^2, \quad \tau_{xy} = \frac{\partial^2 f}{\partial x \partial y} = 0 \tag{9}
\]

\[
\sigma_x = \frac{\partial^2 f}{\partial y^2} = A(X^2 - Y^2), \quad \sigma_y = \frac{\partial^2 f}{\partial x^2} = A(Y^2 - X^2), \quad \tau_{xy} = \frac{\partial^2 f}{\partial x \partial y} = -2AXY \tag{10}
\]

Figure 1. Confining stresses due to \( f_2(x,y) \). Figure 2. Confining stresses due to \( f_4(x,y) \).

Adding together (9) and (10), the stress state (11) is obtained (see fig. 3).

\[
\sigma_x = A(X^2 - Y^2) - B l^2, \quad \sigma_y = A(Y^2 - X^2) - B l^2, \quad \tau_{xy} = -2AXY \tag{11}
\]
Confining stresses vary with A and B, ever enforcing a plain strain state. In a square section confined by square hoops (compression stress positive), the (12) give the increase $\Delta \sigma_z$ of axial stress $\sigma_z$, the (13) and (14) strains and displacements along $Y$ (or $X$, commuting $X$ with $Y$). The concrete mean strain $\varepsilon_{c\text{-mean}}$ of the section side parallel to $Y$ is obtained (15) from (14), for $Y=-l$ and $X=l$ and dividing by $l$. The hoop gives corner and shear reactions (variable stresses along the hoop-arms).

$$
\Delta \sigma_z = \nu (\sigma_x + \sigma_y) = -\nu \left( A(X^2 - Y^2) + B l^2 + A(Y^2 - X^2) - B l^2 \right) = 2\nu B l^2
$$

$$
\varepsilon_y = -\frac{AX^2(v+1) - (AY^2(v+1) + Bl^2(v-1))}{E}; \quad \gamma_{xy} = \gamma_{yx} = -\frac{4(v+1)}{E}AXY
$$

$$
V_y = -\frac{Y\left(3AX^2(v+1) - (AY^2(v+1) + Bl^2(v-1))\right)}{E}
$$

$$
\varepsilon_{c\text{-mean}} = \frac{l^2\left(2A(v+1) + B(1-v)\right)}{E}
$$

For circular hoops on square or circular cross sections, the Airy’s functions expressed in polar coordinates give (16) a uniform radial pressure $q/S$, with $r$=radial direction, $\theta$=orthogonal to $r$ direction, $S$= spacing between hoops. The (17) gives the increase of axial stress $\Delta \sigma_z$ due to confinement. The shear stresses are null and the strains of concrete and hoop are given by (18) and (19), respectively.

$$
\sigma_r = \sigma_\theta = \frac{q}{S}
$$

$$
\Delta \sigma_z = \nu (\sigma_r + \sigma_\theta) = 2\nu \sigma_r = 2\nu \frac{q}{S}
$$

$$
\varepsilon_r = \frac{1}{E} (\sigma_r - \nu \sigma_\theta); \quad \varepsilon_\theta = \frac{1}{E} (\sigma_\theta - \nu \sigma_r) \quad \text{and via (16)} \quad \varepsilon_\tau = \varepsilon_{\tau} = -\frac{q}{S \cdot E} (1 - \nu)
$$

$$
\varepsilon_{\tau} = \frac{q \cdot R}{E_s \cdot A_s}
$$
2.1 Evaluation of the Airy’s coefficients

To quantify confining stresses, the coefficients A and B must be evaluated. For longitudinal bars infinitely stiff, the confining stresses, times the hoop spacing S, give the forces (20) orthogonal to the hoop-arm parallel to X. The axial force on this arm (21) equals the sum of shear stresses acting on the arm considered and of pressures acting on the arm orthogonal to the considered one (see fig.4).

\[ q(x) = \left( A(l^2 - x^2) - B l^2 \right) S \]  \hspace{1cm} (20)

\[ N_a = \frac{S l^3 (A + 3B)}{3} \]  \hspace{1cm} (21)

![Figure 4. Hoop model.](image)

The (22) give mean axial strain and elongation of each arm. The deflection along Y (or X, commuting X with Y) of an arm parallel to X is the sum of deflection due to \( q(x) \) and elongation due to \( N_a \) (23). The (24) give the displacement \( \Delta l_{ch} = (\varepsilon_{c,mean} + \varepsilon) \) of a side of the concrete section parallel to the hoop-arm, while the displacement orthogonal to the hoop-arm (25) is the difference between the unconfined concrete displacement \( (\varepsilon_{c,mean} + \varepsilon) \) and its displacement owing to the confining stresses.

\[ \varepsilon_{c,mean} = \frac{S l^3 (A + 3B)}{3E_c A_s} \quad \text{and} \quad \Delta l_{a} = \frac{S l^4 (A + 3B)}{3E_c A_s} \]  \hspace{1cm} (22)

\[ V_{y,ct} = S \left[ A \left( \begin{array}{c} A X^2 - 15l^2 A_X X^2 (A - B) + 3l^4 A X^2 (9A - 10B) + l^4 (A(120l_b - 13l^2 A_b) + 15B(24l_b + l^2 A_b)) \end{array} \right) \right] \]  \hspace{1cm} (23)

\[ \Delta l_{ch} = \frac{l(2A (u + 1) + B (1 - u))}{E_c} \]  \hspace{1cm} (24)

\[ V_{y,ch} = \frac{-l(3A X^2 (u + 1) - l^2 (A (u + 1) + B (u - 1)) + E_c u \varepsilon)}{E_c} \]  \hspace{1cm} (25)

The compatibility concrete-hoop is imposed by the least squares method (26), (27) in a direction either orthogonal or parallel to each hoop-arm, so obtaining the coefficients A and B of Airy’s function (28), (29). The confining stresses acting on a square section due to a square hoop are immediately evaluated trough (11) and \( \Delta \sigma_3 \) trough (12).

\[ f_{min}(A, B) = \left( \int_0^l V_{y,a} dx - \left[ \int_0^l V_{y,ch} dx \right]^2 \right) + \left[ (l + u l \varepsilon) - \Delta l_{ch} \right] - (l + \Delta l_a)^2 = \min \]  \hspace{1cm} (26)

\[ \begin{bmatrix} \frac{\partial f}{\partial A} \\ \frac{\partial f}{\partial B} \end{bmatrix} = 0 \quad \Rightarrow \quad A, B \]  \hspace{1cm} (27)
In the same way the Airy's coefficient \( q \) and the confining stresses acting on a circular or cross section due to a circular hoop are evaluated (30), being \( R \) the radius of the confined concrete section.

\[
q = \frac{E_z \cdot \varepsilon_z \cdot A_s \cdot \nu \cdot S}{R \cdot E_z \cdot S + E_z \cdot A_s \cdot (1 - \nu) \cdot (\nu \cdot \varepsilon_z + 1)} \tag{30}
\]

In the (1)(30) Poisson's ratio depends on axial strain \( \varepsilon_z \). The law adopted (31) (see Kupfer et oth. 1969) has a limit \( \nu \leq 0.5 \), being \( \nu_0 = 0.2 \) and \( \varepsilon_{c0} = \) strain relevant to unconfined concrete peak stress.

\[
\nu = \nu_0 \left[ 1 + 0.2 \left( \frac{\varepsilon_z}{\varepsilon_{c0}} \right) 0.5 \left( \frac{\varepsilon_z}{\varepsilon_{c0}} \right)^2 + 1.55 \left( \frac{\varepsilon_z}{\varepsilon_{c0}} \right)^3 \right] \tag{31}
\]

3 ANALYTICAL MODEL FOR THE CONFINED CONCRETE

The hoops of a circular section are only axially stressed and apply to the concrete an uniformly distributed radial pressure (16). The confining stress is constant all over the section, so an active confinement model can be adopted for the stress-strain law. The square hoops, by contrast, exert on the confined concrete non-uniform pressures and shear stresses. In any case, the mean radial confining stress is constant (see 3.1) on any circumference of radius \( r \) within the hoop perimeter. So the stress-strain law of the confined concrete can be given by an active confinement model, assuming it as valid in a mean sense. The active confinement model adopted (32) is the triaxial one proposed by Attard and Setunge (1996); it describes analytically the stress-strain relationship of confined concrete (constitutive law) given the relationship of unconfined one (stress-strain law due to Sargin (1971) has been choosen). The stress \( \sigma_z \) and strain \( \varepsilon_z \) are divided by the compression strength of confined concrete \( f_{cc} \) and the relevant strain \( \varepsilon_{cc} \) respectively, so obtaining the equation shown (32).

\[
y = \frac{ax + bx^2}{1 + cx + dx^2} \quad \text{where: } y = \frac{\sigma_z}{f_{cc}} ; x = \frac{\varepsilon_z}{\varepsilon_{cc}} \quad \text{with } \forall x \geq 0 \text{ and } 0 \leq y \leq 1 \tag{32}
\]

In order to define the whole stress-strain law, the model uses two groups of constants \( a, b, c, d \), one group for the ascending part and the other for the descending part of the curve.

3.1 Confinement pressures produced by square hoops

The model adopted is based on tests carried out on cylinders subjected to triaxial stress states. In order to apply it, a cylindrical concrete core and a confining stress acting on it must be identified. For a cylindrical column, the model is immediately applicable, while for a prismatic column of square cross section a cylinder must be identified. This is the reason why, for a square section, it is worth considering the circumferences within the confined core; the expressions previously determined must be transformed in polar co-ordinates, therefore. In the area within the hoop the distribution of confining stresses is given by the (11). Putting \( x = r \cos \phi \), \( y = r \sin \phi \), the (11) is expressed in polar co-ordinates obtaining the (33). The radial stress \( \sigma_x \) and the shear stress \( \tau_{nm} \), on any circumference of radius \( r \) within the confined core (see Fig. 5 for \( r=R \) and \( r=R/2 \)), are given by (34) and (35).
\[ \sigma_{x} = A r^{-2} \left( \cos^2 \phi \cdot \sin^2 \phi \right) - B l^2, \quad \sigma_{y} = - A r^{-2} \left( \cos^2 \phi \cdot \sin^2 \phi \right) - B l^2, \quad \tau_{xy} = - 2 A r^{-2} \sin \phi \cos \phi \]  
(33)

\[ \sigma_{n} = \sigma_{x} \cos^2 \phi + \sigma_{y} \sin^2 \phi + 2 \tau_{xy} \sin \phi \cos \phi = A r^{-2} \cos(4 \phi) - B l^2 \]  
(34)

\[ \tau_{nm} = \sigma_{x} \sin \phi \cos \phi - \sigma_{y} \sin \phi \cos \phi + \tau_{xy} \left( \sin^2 \phi - \cos^2 \phi \right) = 4 A r^{-2} \sin \phi \cos \phi \left( \cos^2 \phi - \sin^2 \phi \right) \]  
(35)

The mean pressure of confinement \( f_{mn} \) on any circumference of radius \( r \) within the hoop perimeter, is given by (36), that is referred, for symmetry, to a quarter of the cross section, only. As said before the mean value \( f_{mn} \) of \( \sigma_{n} \) does not depend from \( r \) while, for \( r = 0 \), \( \sigma_{n} = f_{mn} \) \( \forall \phi \).

\[ f_{mn} \cdot \frac{\pi}{2} = \int_{0}^{\frac{\pi}{2}} \sigma_{n} d\phi \Rightarrow f_{mn} = \frac{2}{\pi} \left( A r^{-2} \left[ \sin(4 \phi) \right]_{0}^{\pi/2} - B l^2 \frac{\pi}{2} \right) = \frac{2}{\pi} \left( 0 - \frac{\pi}{2} B l^2 \right) \]  
(36)

Figure 5. Stress state along two circumferences within the confined core of square section.

The constancy of \( f_{mn} \) is very important; it authorises an active confinement model for prismatic columns. Moreover, the mean value of the confining pressure \( (B l^2) \) agrees with the increase of axial stress \( (\Delta \sigma_{z} = 2 \nu B l^2) \) given by the (12), so strengthening the proposed similarity prismatic/cylindrical.

### 3.2 Distribution of the confinement pressures along the column

The compatibility between longitudinal bars and concrete has been imposed via a least squares best fitting, so obtaining the distribution of the confining stresses along \( z \). In any cross section (parallel to \( \text{x-y} \) plane) the strains on the concrete are assumed to be distributed in the same way above shown for sections containing hoops. The distribution is scaled with \( z \) according to a cubic law having parallel to \( z \) axis tangents either in sections containing hoops or in sections midway between them. Each bar receives by each hoop a force \( N_{sl}/2 \) parallel to the hoop-arm. The orthogonal to \( z \) axis displacement of each longitudinal bar varies between a minimum attained in each section containing a hoop (rigid translation orthogonal to \( z \) produced by the hoop extension) and a maximum attained adding, to the previous one, the flexural displacement. The ratio between the minimum value and the mean value along \( z \) of the displacement above said, gives (37) a coefficient \( k_{sl} \). This coefficient, times the maximum confinement pressure \( f_{mn} \), gives the mean confinement pressure \( f_{r} \) along the column (38).

\[ k_{sl} = \frac{24}{\xi_{l}^{i}} \frac{3}{\beta + 24 \xi_{l}^{i}} \quad \text{where} \quad \xi_{l}^{i} = \frac{\phi_{long}}{S}; \quad \xi_{l} = \frac{\phi_{ml}}{l_{sl}}; \quad \beta = \frac{\phi_{l}}{\phi_{long}} \]  
(37)

\[ f_{r} = k_{sl} f_{mn} \]  
(38)
When the bending stiffness of longitudinal bars can be neglected (low values of \( \xi \)), the confinement pressures are distributed along \( z \) owing to the arching actions between two consecutive hoops. The (39) (Mander et al., 1988) expresses the ratio between the confined section area, midway between two hoops, and the area within the hoop. The coefficient \( k_{sl} \) is also used for circular hoops.

\[
k_i = \left(1 - \frac{S}{4l} \right)^2 \text{ with } k_{sl} \geq k_i
\]  

(39)

### 3.3 Sections confined by overlapped hoops and internal ties

To obtain the confinement relevant to overlapped hoops (see Fig. 6), the results obtained previously for a single hoop, either square or circular, are superimposed. Figure 7 shows two sections with overlapped internal and external hoops; the scheme at left approximates S2, the scheme at right S3 of Fig. 6. The pressure exerted by the external hoop (see fig. 7) is \( f_{re} \), while \( f_{ri} \) is the pressure developed by the internal hoop.

\[
f_{re} = \frac{1}{A} \left( f_{ri} A_i + f_{ri} A_2 \right) = \frac{1}{A} \left( f_{ri} A_i + f_{ri} A \right)
\]

(41)

The last expression in (41) is valid for all the confined by internal and external hoops sections, where \( A \) is the area within the external hoop and \( A_1 \) is the area within the internal hoop. For the (a) scheme and the (b) scheme the (41) gives the (42) and the (43), respectively.

\[
f_r = \frac{1}{2} f_{ri} + f_{re}
\]

(42)

\[
f_r = \frac{\pi}{4} f_{ri} + f_{re}
\]

(43)
4 THE PROPOSED MODEL IN COMPARISONS WITH EXPERIMENTAL RESULTS

The proposed model has been compared with experimental data relevant to columns of circular and square cross section. The results relevant to the overlapped hoops (Scott et al., 1982) and circular hoops (Ahmad & Shah, 1982) are shown in Table 1, Table 2 and Figure 9 (Ks is the ratio between confined and unconfined concrete peak stresses, \( \varepsilon_{cc}/\varepsilon_{c0} \) is the ratio between confined and unconfined concrete strains corresponding to the peak stresses). Figure 8 shows, for test C-6 (Scott et al., 1982), a comparison among the experimental and analytical curves (\( \sigma-\varepsilon \)) relevant to the proposed model and models proposed by other authors. The obtained results have confirmed the validity of the proposed model.

Table 1. Experimental and Analytical Results (Scott et al, 1982)

<table>
<thead>
<tr>
<th>Column Type</th>
<th>( f_{cc}/f'_{c} )</th>
<th>( \varepsilon_{cc}/\varepsilon_{c0} )</th>
<th>Proposed Model</th>
<th>( \varepsilon_{cc}/\varepsilon_{c0} )</th>
<th>Comparisons</th>
<th>( % ) (Error %) (Error %)</th>
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<tr>
<td>2 S3</td>
<td>1.24</td>
<td>2.889</td>
<td>2.778</td>
<td>5.23%</td>
<td>-3.85%</td>
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<tr>
<td>3 S3</td>
<td>1.54</td>
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<td>1.54</td>
<td>2.361</td>
<td>-0.10%</td>
<td>6.25%</td>
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<tr>
<td>6 S2</td>
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<td>2.500</td>
<td>1.23%</td>
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<tr>
<td>7 S2</td>
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<td>1.46</td>
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<tr>
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<td>-10.41%</td>
<td>16.17%</td>
</tr>
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<td>6.25%</td>
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<tr>
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<td>1.57</td>
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<td>-1.78%</td>
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<td>2.219</td>
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<tr>
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<td>24.46%</td>
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<td>3.306</td>
<td>3.86%</td>
<td>48.75%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-2.42%</td>
<td>16.37%</td>
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Figure 8. Experimental and Analytical Stress Strain curves (Column C-6, Scott et al, 1982).
Table 2. Comparisons of experimental results (Ahmad & Shah, 1982) with the values predicted by models

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experimental values</th>
<th>Mander et al.</th>
<th>Park et al.</th>
<th>Ahmad et Shah</th>
<th>Elnashai et al.</th>
<th>Proposed model</th>
</tr>
</thead>
<tbody>
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<td>$f_{cc}/f_{c}$</td>
<td>$\varepsilon_{cc}/\varepsilon_{c0}$</td>
<td>$f_{cc}/f_{c}'$</td>
<td>$\varepsilon_{cc}/\varepsilon_{c0}$</td>
<td>$f_{cc}/f_{c}'$</td>
<td>$\varepsilon_{cc}/\varepsilon_{c0}$</td>
</tr>
<tr>
<td>II / 1</td>
<td>1.205</td>
<td>1.905</td>
<td>1.574</td>
<td>3.857</td>
<td>1.245</td>
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<td>1.420</td>
<td>3.091</td>
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<td>5.000</td>
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<td>1.333</td>
<td>1.517</td>
<td>3.600</td>
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Total (Error %) 30.38% 83.27% 4.56% -37.94% 6.17% 30.74% 5.74% 24.71% 1.94% 9.31%

Figure 9. Comparisons of experimental results (Ahmad & Shah, 1982) with the values predicted by models.

5 CONCLUSIONS

An obvious utilisation of the results obtained should be in design or in codes of practice. The first comparison between provisions of theory and experimental results shows a very good agreement. The proposed model of confinement, being analytical, allows parametric studies and an optimisation of the transversal reinforcements. In particular, any detail of the transversal bars as difference between single and overlapped hoops, influence of hoop spacing, influence of ratio between flexural and axial stiffness of hoops and so on, can be explored.

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REFERENCES


